RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SECOND SEMESTER EXAMINATION, MAY 2023

FIRST YEAR [BATCH 2022-25]

Date : 24/05/2023 **PHYSICS [Honours]**

Time : 11 am – 1 pm **Paper : CC3** Full Marks : 50

Answer **any five** questions: [5×10]

1. a) Derive the complex form of Fourier series and find that the expression for the co-efficient.

   b) $f(x)=\begin{cases} x & \text{in } (0,\pi) \\ 2\pi - x & \text{in } (\pi, 2\pi) \end{cases}$

   Find the Fourier series for $f(x)$ hence deduce $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$.

   c) The turning moment T is given for a series of values of the crack angle;

θ:	0^o	30^o	60^o	90^o	120^o	150^o	180^o
T:	0	5224	8097	7850	5499	2626	0

   Obtain the first four terms in a series of Sine to represent T and calculate T for $\theta = 75^o$. [2+4+4]

2. a) If Fourier transform of function $f(x)$ is $g(s)$, then show that Fourier transform of $f(x)\cos ax$ is $\frac{1}{2}\left[g(a+s)+g(a-s)\right]$.

   b) Find the Fourier transform of the function $f(x)=\delta(x-b)$ where $\delta(x)$ is Dirac-delta function and b is some constant.

   c) Show that the Fourier transform of $f(x)$ where $f(x)=\begin{cases} 1 \text{ for } |x|<0 \\ 0 \text{ for} |x|>a>0 \end{cases}$ is given by $F(s)=\sqrt{\frac{2}{\pi}}\frac{\sin as}{s}$. Hence using Perseval's identity, show that $\int_0 \left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$. [2+3+(3+2)]

3. a) Check the following series diverging or converging

   (i) $1+\frac{1}{2!}+\frac{1}{4!}+\cdots$

   (ii) $1+\frac{1}{3!}+\frac{1}{5!}+\cdots$

   Find the value of $\left(1+\frac{1}{2!}+\frac{1}{4!}+\cdots\right)^2-\left(1+\frac{1}{3!}+\frac{1}{5!}+\cdots\right)^2$.

   b) Discuss the convergence of the following series: $1-\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{3}}-\frac{1}{\sqrt{4}}+\frac{1}{\sqrt{5}}-\frac{1}{\sqrt{6}}+\cdots$.

   c) A die is thrown 8 times. Find the probability that '5' will show (i) exactly twice, (ii) at least seven times (iii) at least once. [(1+1+2)+3+3]

4. a) Show that Poisson distribution is a limiting case of binomial distribution.

   b) Students of a class were given an aptitude test. Their marks were found to be normally distributed with mean 60 and standard deviation 5. What percentage of students scored:

   (i) more than 60 marks

   (ii) less than 56 marks

   (iii) between 45 and 65 marks

Given: $P(x)=\frac{1}{\sigma\sqrt{2n}}e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$, $P(0.8)=0.2881$, $P(1)=0.3413$, $P(3)=0.4986$

$P(x)$ is density function of normal distribution.

c) The random variable $x(-\infty<x<\infty)$ is distributed according to normal distribution $P(x)=\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{x^2}{2\sigma^2}\right)$, find the probability density of the random variable $y=x^2$. [3+3+4]

5. a) Show that the ODE is not self adjoint.

   b) Find the weight factor $w(x)$ and write ODE in a form $\bar{L}y(x)=nw(x)y(x)$ such that itr becomes self-adjoint.

   c) Write the inner product for two eigenfunctions of ODE including the boundary term.

   d) What interval $[a,b]$ should we choose so that two different eigenfunctions are orthogonal? [2+3+3+2]

6. a) Generating function of Legendre Polynomials $P_n(x)$ is $g(x,t)=\frac{1}{\sqrt{1-2xt+t^2}}=\sum_{n=0}^{\infty}P_n(x)t^n$.

   Using this find $P_0(x), P_1(x)$.

   b) Derive the recurrence relation: $(2n+1)xP_n(x)=(n+1)P_{n+1}(x)+nP_{n-1}(x)$. [4+6]

7. Answer the following questions:

   a) By changing the coordinates appropriately simplify the following PDE and solve it.

   $\frac{\partial\psi}{\partial x}+2\frac{\partial\psi}{\partial y}+(2x-y)\psi=0$

   b) Cuylindrical coordinates are related to Cartesian coordinates are related by:

   $x=\rho\cos\phi$

   $y=\rho\sin\phi$

   $z=z$

   Express the operators $\frac{\partial}{\partial x},\left(\frac{\partial}{\partial x}\right)^2$ in cylindrical coordinates. [4+6]

8. a) Suppose the following differential equation refers to a problem of two dimensional steady flow of heat:

$$\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}=0$$

   Solve for T with the following boundary conditions:

   $T(0,y)=0;\ T(x,\alpha)=0$

   $T(a,y)=0;\ T(x,0)=\sin\left(\frac{\pi x}{a}\right)$

   b) Express the following integral in term of gamma function $\int_0^1 x^4\left[\ln\left(\frac{1}{x}\right)\right]^3 dx$.

   c) Show that $\beta\left(m,\frac{1}{2}\right)=2^{2m-1}\beta(m,mn)$. [6+2+2]

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